Math 2374 Syllabus

- How to get started
- Assessment: quizzes, Mathematica notebooks, 3 midterm exams, final exam.
- Put the exam dates on your calendar.
- No homework to hand in. Quizzes and exams are based on the homework.
- Missing exams and quizzes
- Course content is organized around Modules
- Mathematica notebooks, downloading Mathematica. Get to work on the notebooks before the session on Thursday.

There is too much to read!

Section 1.3: Matrices, determinants and the cross product. equations of planes, areas of parallelograms.

We learn

- Determinants of 2x2 and 3x3 matrices (nxn in section 1.5)

$$= -\mathrm{Det}\begin{bmatrix} b & a \\ d & c \end{bmatrix} = -(bc-ad)$$

$$\mathrm{Det}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad-bc$$

$$\mathrm{Det}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = 1\cdot 4 - 2\cdot 3 = -2$$

$$\mathrm{Det}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = aei + bfg + cdh - gec - hfa - idb$$

$$\mathrm{Det}\begin{bmatrix} 1 & 1 & 0 \\ 2 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix} = \begin{matrix} -2+0+0 \\ -0-0-4 \end{matrix} = -6$$

Determinants satisfy:
Linearity in each row and in each column
If we interchange two rows (or two columns) the determinant is multiplied by -1.
If we add a multiple of one row to another, the determinant is unchanged. (Same for columns).
If two rows (or two columns) are the same, the determinant is 0

$$\mathrm{Det}\begin{bmatrix} a & b \\ a & b \end{bmatrix} = ab - ab = 0$$

$$= \mathrm{Det}\begin{bmatrix} a & b \\ a-a & b-b \end{bmatrix} = \mathrm{Det}\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix}$$

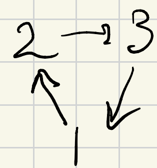
$$= 0$$

The cross product. This works for vectors in $\mathbb{R}^3$. $\mathbb{R}$ = real numbers

Definition:

Given two vectors $a = (a_1, a_2, a_3)$, $b = (b_1, b_2, b_3)$, the cross product is a "cross" b

$$a \times b = (a_2 b_3 - a_3 b_2,\ a_3 b_1 - a_1 b_3,\ a_1 b_2 - a_2 b_1)$$



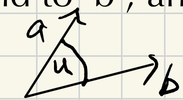
$$= \operatorname{Det}\begin{bmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}$$

where

$i = (1,0,0)$, $j = (0,1,0)$, $k = (0,0,1)$

Properties of the cross product:

$a \times b$ is perpendicular to a and to b, and has length $||a||\,||b||\,|\sin u|$



a, b, $a \times b$ is a right-handed set of vectors.

To show $a \times b$ is perp. to a:

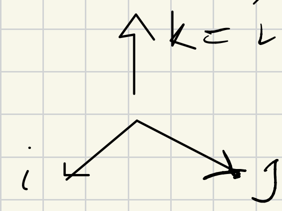
$$(a \times b) \cdot a = \operatorname{Det}\begin{bmatrix} a_1 & a_2 & a_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}$$

$= 0$ (2 rows the same).

Example $(1,0,0) \times (0,1,0)$

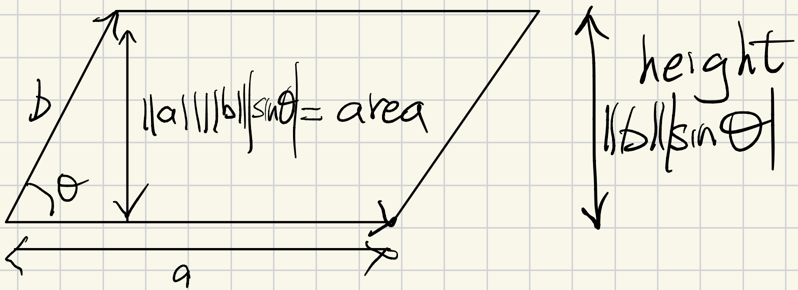
$= (0,0,1)$

$k = i \times j$



Determinants, cross products, areas of parallelograms and volumes of parallelepipeds

In 3-D, a x b has length equal to the area of the parallelogram determined by a and b.

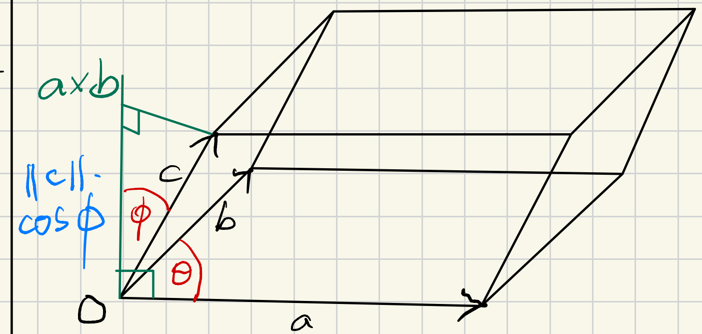


$$a \times b = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$(a \times b) \cdot c = \begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

In 3-D the volume of the parallelopiped determined by a , b and c is the triple scalar product (a x b) • c (in absolute value).

This equals the determinant of the matrix {a|b|c}.



$$\text{Volume} = \|a\|\,\|b\| \sin\theta \, \|c\| \cos\phi$$

$$= \text{base area} \cdot \text{height}$$

$$= \big|\,\|a\|\,\|b\| \sin\theta\,\big| \cdot \|c\| \cos\phi$$

Pre-Class Warm-Up !!

- Evaluate the cross product

$(1, -1, 2) \times (-1, 2, 1)$

Answer:

a. $(5, 3, -1)$

b. $(-1, -2, 2)$

c. $(-5, -3, 1)$

d. $(1, -2, -2)$

e. None of the above

The quiz on Thursday next week will only be on sections 1.3 and 1.5 of the book
(In the schedule it says 2.1 as well.)

Why does the triple product $(a \times b) \bullet c$ equal Det $\{a|b|c\}$?
This means we don't really need the triple product.

Summary of parallelepipeds:
In every dimension the volume of the parallelepiped determined by n vectors equals the absolute value of the determinant of the matrix they form.
Dimension 3: we have seen it, and it's on page 40
Dimension 2: it's deduced on page 39

Also we have seen that the area of the parallelogram determined by 2 vectors in 3-D is the length of the cross product.

Equations of planes, distance from a point to a plane, line of intersection of two planes, etc.

You have HW questions like:

Find an equation of the plane that ...is perpendicular to $(1,-1,2)$ that passes through $(0,1,3)$

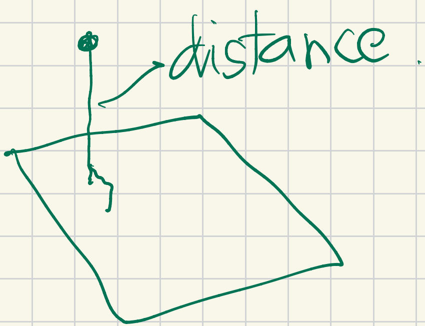
It is $x - y + 2z = D$, $0 - 1 + 6 = D$

$D = 5$.

$(1,2,3)$

Find the distance from the point to the plane

Find the intersection of the two planes



See the second video.

Page 41

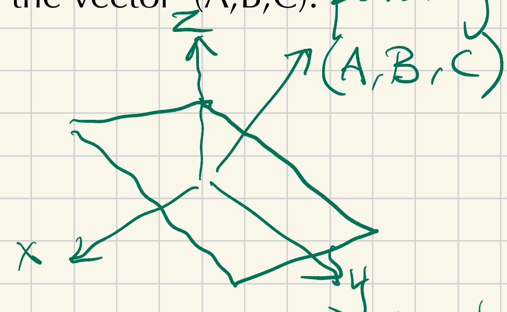
The equation of a plane in 3-space has form

$Ax + By + Cz + D = 0$

or $Ax + By + Cz = D$

Why?

$Ax + By + Cz = 0$ is the plane perpendicular to the vector (A,B,C). passing through $(0,0,0)$



$Ax+By+Cz=0$ is the condition that (x,y,z) is perp. to (A,B,C)

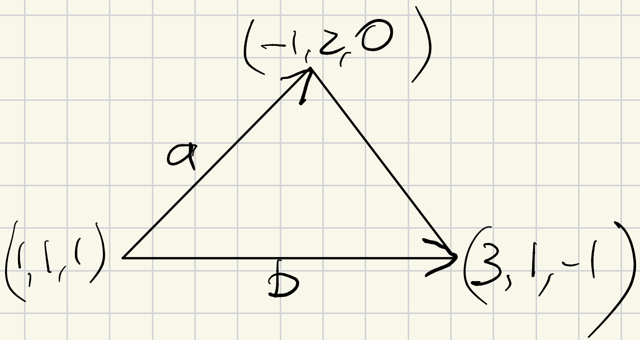
Example: Find the equation of the plane perpendicular to $(1,-2,3)$ passing through $(-1,0,2)$.

Possible answers:

1. $-x + 2z = 2$
2. $x - 2y + 3z = 5$ ✓
3. $x - 2y + 3z = 1$
4. $x - y + 2z = 3$
5. None of the above -

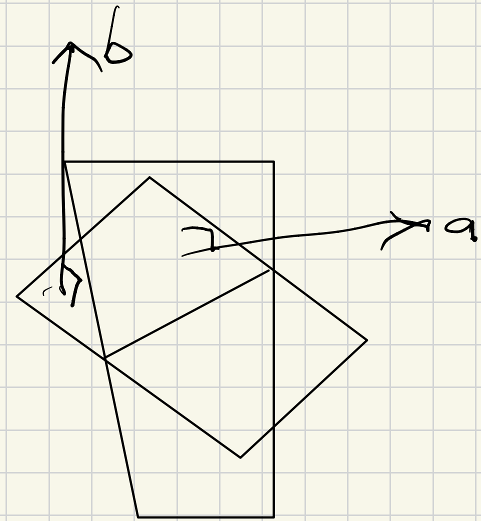
Like page 42 Example 11

Find the equation of the plane passing through the points
(1,1,1), (-1,2,0) and (3,1, -1)



$a \times b$ is perpendicular to the plane

Proceed as before.



$a \times b$ points in the direction of the line of intersection

Find a point on the line.

Like page 50 qn 20: Find the intersection of two planes.

Like page 43

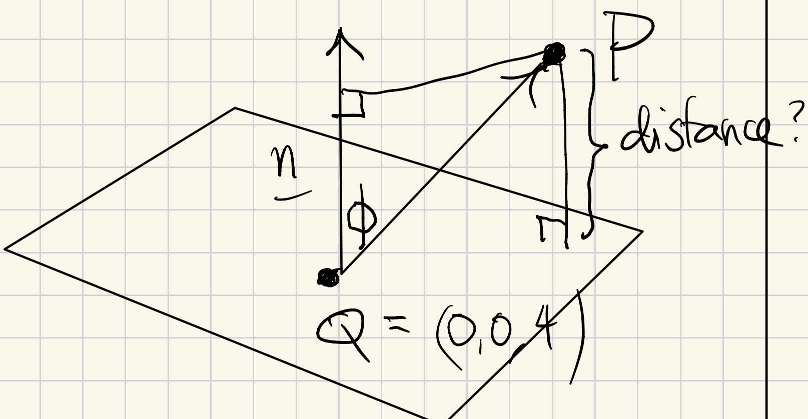
Find the distance from the point (1,0,-1) to the plane $2x - y + z = 4$

First: which of the following points lie in the plane?

1. (1,0,-1)
2. (1,0,2)
3. (1,1,1)

(0,0,4) lies in plane.

Could you find another point in the plane?



$$\underset{\sim}{n} \cdot (P - Q) = \|n\| \, \|P - Q\| \cos\phi$$

$$\text{distance} = \|P - Q\| \cos\phi$$

$$= \frac{\underset{\sim}{n} \cdot (P - Q)}{\|n\|}$$